Section 11.8 Solutions math 31

6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$ Ratio Test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{x^n} \right| =$

$|x| \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^2 = |x| < 1$, so $R=1$

at $x=-1$, $\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by Integral Test

at $x=1$, $\sum_{n=1}^{\infty} \frac{(-1)^n(1)^n}{n^2} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ converges by Alternating Series Test

Interval of convergence: $-1 \le x \le 1$

10. $\sum_{n=1}^{\infty} \frac{10^n x^n}{n^3}$; Ratio Test: $\lim_{n\to\infty} \left| \frac{10^{n+1} x^{n+1}}{(n+1)^3} \cdot \frac{n^3}{10^n x^n} \right| =$

$|10x| \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^3 = |10x| < 1$, so $R = \frac{1}{10}$

at $x = -\frac{1}{10}$, $\sum_{n=1}^{\infty} \frac{(10)^n(-\frac{1}{10})^n}{n^3} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^3}$ converges by Alternating Series Test

at $x = \frac{1}{10}$, $\sum_{n=1}^{\infty} \frac{(10)^n(\frac{1}{10})^n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^3}$ converges by the Integral Test

Interval of Convergence: $-\frac{1}{10} \le x \le \frac{1}{10}$

12. $\sum_{n=1}^{\infty} \frac{x^n}{n3^n}$ Ratio Test: $\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{x^n} \right| =$

$\left|\frac{x}{3}\right| \lim_{n\to\infty} \left(\frac{n}{n+1}\right) = \left|\frac{x}{3}\right| < 1$, so $R=3$

at $x=-3$, $\sum_{n=1}^{\infty} \frac{(-3)^n}{n(3)^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges by Alternating Series Test

at $x=3$, $\sum_{n=1}^{\infty} \frac{3^n}{n3^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by Integral Test

$\therefore$ Interval of Convergence: $-3 \le x < 3$

18. $\sum_{n=1}^{\infty} \frac{n}{4^n}(x+1)^n$ Ratio Test: $\lim_{n\to\infty} \left| \frac{(n+1)(x+1)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{n(x+1)^n} \right| =$

$\left|\frac{x+1}{4}\right| \lim_{n\to\infty} \left(\frac{n+1}{n}\right) = \left|\frac{x+1}{4}\right| < 1$, or $|x+1| < 4$, $R=4$

at $x=-5$, $\sum_{n=1}^{\infty} \frac{n(-5+1)^n}{4^n} = \sum_{n=1}^{\infty} (-1)^n n$ Diverges by Alternating Series Test

at $x=3$, $\sum_{n=1}^{\infty} \frac{n(3+1)^n}{4^n} = \sum_{n=1}^{\infty} n$ Diverges by the Divergence Test

$\therefore$ Interval of Convergence: $-5 < x < 3$